Simplified Load Combination Rules
Economic Aspects and Reliability

François Colling, Michael Mikoschek
University of Applied Sciences Augsburg

1. Introduction

Finding the decisive design load for timber constructions according to the rules of Eurocode 0 [1] and Eurocode 5 [2] is time consuming. In addition, the effort is often unnecessary regarding the (small) differences in the resulting values. Therefore the following simplification was suggested in [3] for loads acting in the same direction.

Firstly the impact (forces, moments) of the two design loads $e_{d,1}$ and $e_{d,2}$ have to be calculated by using the following equations

$$e_{d,1} = 1{,}4 \cdot \left(g_k + \sum q_{k,i}\right) \tag{1}$$

and

$$e_{d,2} = 1{,}35 \cdot g_k + 1{,}5 \cdot q_{k,1} \tag{2}$$

with

g_k	=	characteristic dead load
$q_{k,1}$	=	characteristic "leading" live load
$q_{k,i}$	=	further characteristic live loads.

Dividing these values by the corresponding k_{mod} - values leads to the decisive load combination:

$e_{d,1}/max\ k_{mod} > e_{d,2}/k_{mod,Q1}$ → $e_{d,1}$ is decisive!

$e_{d,2}/k_{mod,Q1} > e_{d,1}/max\ k_{mod}$ → $e_{d,2}$ is decisive!

The proposed simplification has been compared with the "accurate" rules according to the Eurocodes and the differences have been found to be small and mostly on the safe side under the following conditions (see [3]):

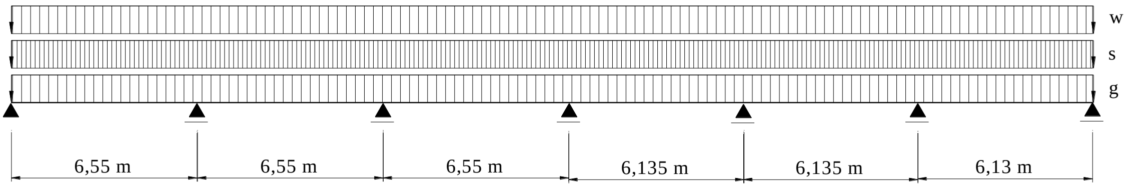
- Ratio $g_k/(g_k+\Sigma q_{k,i}) \leq 0{,}6$.
- Building categories A, B, C and D (not E with load duration = long).
- Loads acting in the same direction.

The authors were encouraged to perform further calculations and verifications, especially in view of aspects concerning economy and reliability. The results are shown below.

2. Economic Aspects

In most cases the simplified load combinations lead to higher values for design loads compared to the rules of the Eurocodes. In order to estimate if this results in larger cross sections and thus higher costs, existing timber building elements from projects in [4] were reviewed and redesigned by using the simplified load combinations. These calculations were performed by using the given dimensions (cross sections) of the elements.

Example 1: Top chord of a gable structure



Cross section b/h = 140/360 mm, GL28c

Loads: g_k = 1,60 kN/m (permanent), s_k = 5,70 kN/m (short), w_k = 1,60 kN/m (short)

Leading live load = snow

- Load combinations according to **Eurocodes** (the decisive load combination is marked in bold letters):

  $1{,}35 \cdot g_k$

  $1{,}35 \cdot g_k + 1{,}5 \cdot s_k$

  $1{,}35 \cdot g_k + 1{,}5 \cdot w_k$

  $\mathbf{1{,}35 \cdot g_k + 1{,}5 \cdot s_k + 0{,}6 \cdot 1{,}5 \cdot w_k}$

  $1{,}35 \cdot g_k + 1{,}5 \cdot w_k + 0{,}5 \cdot 1{,}5 \cdot s_k$

- Load combinations according to **simplified rules**:

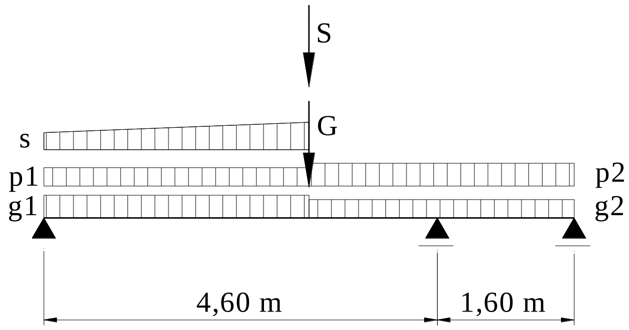
  $1{,}35 \cdot g_k + 1{,}5 \cdot s_k$

  $\mathbf{1{,}4 \cdot (g_k + s_k + w_k)}$

The decisive performance ratios are given below.

Example	rules of Eurocodes			simplified rules		
	Number of load comb.	performance ratio		Number of load comb.	performance ratio	
		bending	shear		bending	shear
1	**5**	**0,85**	**0,79**	**2**	**0,92**	**0,82**

Example 2: Floor joist of an office building with apartment



Cross section b/h = 120/220 mm, C24

Loads: $g_{1,k}$ = 1,156 kN/m, $g_{2,k}$ = 1,114 kN/m, G_k = 2,615 kN (permanent),

$p_{1,k}$ = 0,626 kN/m, $p_{2,k}$ = 1,168 kN/m (medium),

s_k = 0,826 → 1,238 kN/m, S_k = 1,155 kN (short)

Note: A second set of loading (2b) was investigated, but not shown here in detail.

Leading live load is not obvious → p and s are considered each (separately) as leading load.

Load combinations performed on basis of internal reactions (shear forces and moments).

- Load combinations according to **Eurocodes** (the decisive load combination is marked in bold letters):

  $1{,}35 \cdot g_k$

  $1{,}35 \cdot g_k + 1{,}5 \cdot s_k$

  $1{,}35 \cdot g_k + 1{,}5 \cdot p_k$

  $\mathbf{1{,}35 \cdot g_k + 1{,}5 \cdot s_k + 0{,}6 \cdot 1{,}5 \cdot p_k}$

  $1{,}35 \cdot g_k + 1{,}5 \cdot p_k + 0{,}5 \cdot 1{,}5 \cdot s_k$

- Load combinations according to **simplified rules**:

  $1{,}35 \cdot g_k + 1{,}5 \cdot s_k$

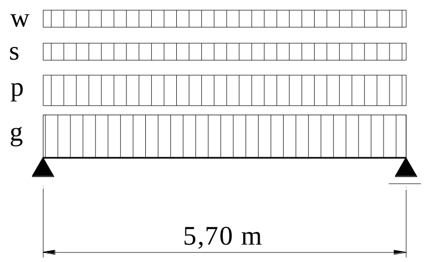
  $1{,}35 \cdot g_k + 1{,}5 \cdot p_k$

  $\mathbf{1{,}4 \cdot (g_k + s_k + p_k)}$

The decisive performance ratios are given below.

Example	rules of Eurocodes			simplified rules		
	Number of load comb.	performance ratio		Number of load comb.	performance ratio	
		bending	shear		bending	shear
2a	**5**	**0,65**	**0,40**	**3**	**0,68**	**0,42**
2b	**5**	**0,93**	**0,65**	**3**	**1,00**	**0,69**

Example 3: Binding beam of a day care facility for children (building category C → live load = short).
Cross section b/h = 240/800 mm, GL28h



Loads: g_k = 46,65 kN/m (permanent),

p_k = 25,54 kN/m (short) → leading live load

s_k = 4,97 kN/m (short) resulting from roof structure transmitted by a wall on the beam

w_k = 0,62 kN/m (short) resulting from roof structure

- Load combinations according to **Eurocodes** (the decisive load combination is marked in bold letters):

$1{,}35 \cdot g_k$

$1{,}35 \cdot g_k + 1{,}5 \cdot s_k$

$1{,}35 \cdot g_k + 1{,}5 \cdot p_k$

$1{,}35 \cdot g_k + 1{,}5 \cdot w_k$

$1{,}35 \cdot g_k + 1{,}5 \cdot s_k + 0{,}7 \cdot 1{,}5 \cdot p_k$

$1{,}35 \cdot g_k + 1{,}5 \cdot s_k + 0{,}6 \cdot 1{,}5 \cdot w_k$

$1{,}35 \cdot g_k + 1{,}5 \cdot p_k + 0{,}5 \cdot 1{,}5 \cdot s_k$

$1{,}35 \cdot g_k + 1{,}5 \cdot p_k + 0{,}6 \cdot 1{,}5 \cdot w_k$

$1{,}35 \cdot g_k + 1{,}5 \cdot w_k + 0{,}5 \cdot 1{,}5 \cdot s_k$

$1{,}35 \cdot g_k + 1{,}5 \cdot w_k + 0{,}7 \cdot 1{,}5 \cdot p_k$

$1{,}35 \cdot g_k + 1{,}5 \cdot s_k + 0{,}7 \cdot 1{,}5 \cdot p_k + 0{,}6 \cdot 1{,}5 \cdot w_k$

$\mathbf{1{,}35 \cdot g_k + 1{,}5 \cdot p_k + 0{,}5 \cdot 1{,}5 \cdot s_k + 0{,}6 \cdot 1{,}5 \cdot w_k}$

$1{,}35 \cdot g_k + 1{,}5 \cdot w_k + 0{,}5 \cdot 1{,}5 \cdot s_k + 0{,}7 \cdot 1{,}5 \cdot p_k$

- Load combinations according to **simplified rules**:

$1{,}35 \cdot g_k + 1{,}5 \cdot p_k$

$\mathbf{1{,}4 \cdot (g_k + p_k + s_k + w_k)}$

The decisive performance ratios are given below.

Example	rules of Eurocodes			simplified rules		
	Number of load comb.	performance ratio		Number of load comb.	performance ratio	
		bending	shear		bending	shear
1	**13**	**0,86**	**0,95**	**2**	**0,91**	**0,97**

From these calculations it can be deduced that

- In all cases the performance ratios by using the simplified load combinations are 3 - 7% higher compared to the rules of Eurocodes. This shows that the simplified rules are on the save side.
- In no case higher cross sections would have been necessary when applying the simplified rules.
- Furthermore, the smaller number of necessary load combinations underline the advantage of the simplified rules.

For a more general assessment further comparing design calculations are necessary, especially for elements with loads acting in different directions (vertical and horizontal loads).

3. Reliability Aspects

The authors have been asked to check the impact of the simplified rules on the reliability. As this topic represents a rather new (and unknown) area to the authors, this request posed a considerable challenge.

The following calculations have been performed by using the EXCEL-tool "CodeCal" [6]. This tool calculates the reliability index β related to a reference period of one year by using defined limit state functions. In case of two live loads this tool uses three limit state functions and a Borges-process to describe the combination of live loads. Unfortunately these functions/rules cannot be modified so that the comparison between the rules of Eurocodes and the proposed simplification can only be presented within the rules used in "CodeCal".

The calculations have been performed by using the distribution parameters given in **Table 1**.

Table 1: Basic Variables used in the reliability analysis

Basic variable	Distribution	Mean μ	Standard dev. σ	Char. %
Resistance R	Lognormal	1,0	0,15	5
Uncertainty ξ	Lognormal	1,0	0,1	---
Dead load G	Normal	1,0	0,1	50
1st live load Q_1	Gumbel	1,0	0,4	98
2nd live load Q_2	Gumbel	1,0	0,7	98

The safety factors for material and loads as well as the load combination factors ψ_0 used are given in **Table 2**.

Table 2: Factors used in the reliability analysis

Factor	Eurocode	Simplified rules	
		Eq.1	Eq.2
γ_m	1,3	1,3	1,3
γ_G	1,35	1,4	1,35
γ_Q	1,5	1,4	1,5
$\psi_{0,1}$	0,7	1,0	---
$\psi_{0,2}$	0,5	1,0	---

The results are shown in the following graph by using the following variables:

β = safety index

α_G = proportion of dead load compared to total load

$$= \frac{G}{G+Q_1+Q_2}$$

α_Q = proportion of the 1st live load to the total of live loads

$$= \frac{Q_1}{Q_1+Q_2}$$

The calculated reliability index is compared to the following target valus:

- β = 3,8
  according to Eurocode 0 for reliability class 2 (RC 2) and buildings in consequence class 2 (CC 2) related to a reference period of 50 years.
  This value is referred to in other calculations, i.e. [7]. This reliability index corresponds to a probability of failure of $P_f = 7{,}2 \cdot 10^{-5}$ approx.
  Note: For a reference period of one year, EC 0 recommends a safety index of β = 4,7 !

- β = 4,2
  according to JCSS [5] for moderate consequences of failure related to one year reference period.
  This reliability index corresponds to a probability of failure of $P_f = 1{,}3 \cdot 10^{-5}$ approx.

From these graphs it can be seen, that the reliability indexes for the simplified equations (1) and (2) are mostly higher than in case of using the load combination rules according to the Eurocodes.

This confirms the results stated in in chapter 2.

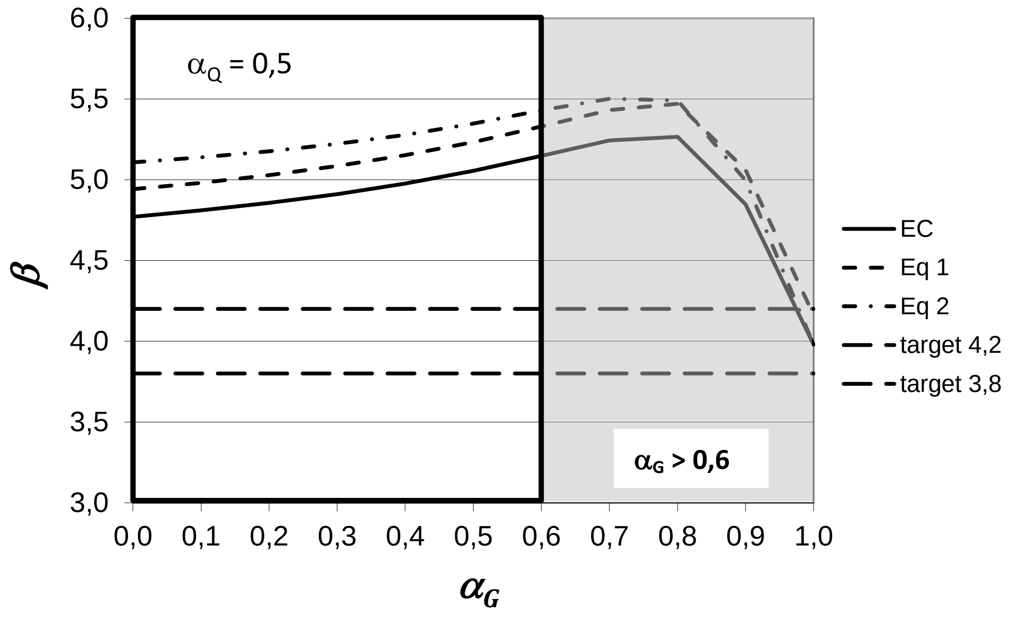


Figure 1: Reliability analysis for α_Q = 0,5

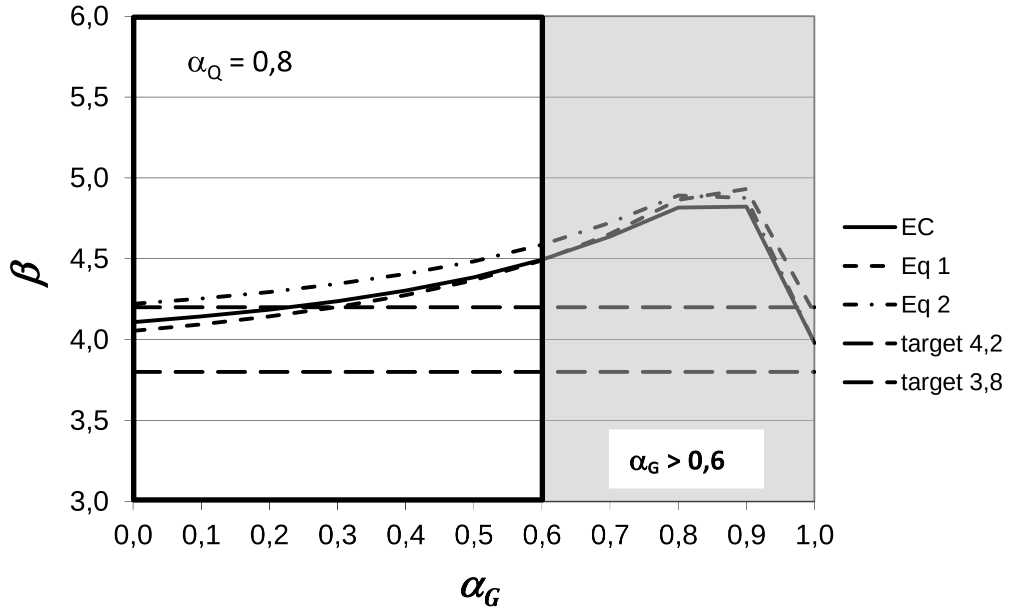


Figure 2: Reliability analysis for α_Q = 0,8

Discussion:

- The calculations performed until now only consider the maximum design loads. They do not consider the influence of k_{mod} which is of fundamental significance for timber structures. Therefore the presented calculations can only be seen as first estimations.
- For a more detailed investigation of the influence of k_{mod} further reliability calculations have to be performed by using appropriate limit state functions.

4. Summary and Outlook

Based on proposed simplified rules for load combination further calculations have been performed with special focus on economic and reliability aspects.

Redesigning three existing building elements by using the simplified rules showed higher performance ratios but did not lead to higher dimensions of the cross sections. This indicates that the simplification lies on the safe side.

First calculations regarding reliability aspects showed that the simplified rules lead to higher reliability indexes compared to the rules of Eurocodes. Further comparing calculations are necessary, especially for elements with loads acting in different directions (vertical and horizontal loads).

Referring to reliability analysis further studies are necessary to consider the influence of k_{mod}. For this the authors have to deepen their knowledge in this field. This could be achieved with COST activities, i.e. training schools or Short Term Scientific Missions (STMS).

Finally there is a need for discussion concerning the based level of safety index β (uniform value for all materials and related reference period) and k_{mod} (simplified set of values).

5. References

[1] EN 1990 (Eurocode 0)

[2] EN 1995-1-1 (Eurocode 5)

[3] Colling F., Mikoschek M. (2015): Load Combinations – State of the Art and Proposals for Simplifications, COST FP1402 2nd Workshop, Pamplona

[4] Harrer Ingenieure (2015): Abschlussbericht PRB Projektgruppe 1 – Erprobung der Praxistauglichkeit der Vorschläge zu EN 1990, EN 1991-1-3 und EN 1991-1-4 – Vergleich Holzbau, Karlsruhe

[5] Joint Committee on Structural Safety JCSS (2001): Probabilistic Model Code, Part 1 – Basis of Design

[6] JCSS, CodeCal - Reliability based code calibration, 2004

[7] Frank Breinlinger, Wolfram Jäger, 2015: Verbesserung der Praxistauglichkeit der Baunormen durch pränormative Arbeit, Teilantrag 1: Sicherheitskonzept und Einwirkungen, Forschungsbericht F 2957 der Forschungsinitiative ZukunftBau