SearchWing - Search Area Calculations

Compute the achievable search area of the searchwing drone and compare achievable areas with the Sea-Watch 3 vessel and the Moonbird plane

Prof. Dr.-Ing. Friedrich Beckmann

January 2019

Searching for people in distress in the mediterranean sea is today done via observation from the search vessels, e.g. Sea-Watch 3 with binoculars and radar or with searching with a plane which is based in Malta (Moonbird). This report calculates the achievable search areas for a vessel and a plane and compares them with the search areas than can be searched with the SearchWing unmanned aerial vehicle.

1 Introduction

In 2016 over 5000 humans died in the mediterranean sea when they tried to reach Europe by sea. Private search and rescue organizations like Sea-Watch, Sea-Eye and ResQShip operate vessels to find ships in distress on the mediterranean sea to rescue humans.

Figure 1: Search area in the central mediterranean sea (Mapsource: OpenStreetMap)

Figure 1 depicts the search area where boats typically are found in distress on the route from Libya to Europe. The distance from Libya to Lampedusa is around 300km and the distance to Malta is 400km. The distance from Malta to the search area is around 300km (not depicted).

2 Assumptions

I compare search operations with parameters according to the International Aeronautical and Maritime Search and Rescue (IAMSAR) Manual [1].

- Object to search is a six person liferaft
- Visibility is 10 nautical miles
- Wind is below 28 km/h, Seawaves 0-1m

Table 1: Sweep widths W for vessels and planes in km

Vehicle	Sweep Width in km	Source
Merchant Vessel	9,3	IAMSAR Table N-4
Plane at altitude 600m	4,3	IAMSAR Table N-6

3 Sea Watch 3

Table 2: Technical parameter of the Sea Watch 3 vessel

Parameter	Value
length	50,35m
width	11,58m
draft	3,56m
Motor	2 x 845 kW
Maximum speed	19 km/h

The typical consumption of a Diesel engine is 200 g/kWh. Therefore we need $2 \cdot 845kW \cdot 0,2kg/kWh = 338kg$ Diesel per hour at full power. The power consumption of the vessel is a cubic function of the vessel speed. As the drag power is proportional to the velocity

$$P = F \cdot v = \frac{1}{2} c_W \rho A v^2 \cdot v = \frac{1}{2} c_W \rho A v^3 \tag{1}$$

The Sea-Watch 3 has a maximum speed of 19 km/h. I assume a cruise speed v of 10 km/h because then the power consumption is only 15-30% of the consumption at full speed. The covered search area is therefore

$$A = Wvt \Rightarrow \frac{A}{t} = Wv = 9,3km \cdot 10km/h = 93km^2/h \tag{2}$$

$93km^2$ per hour according to equation 2. Assuming that search can only be done during daylight I assume ten hours daylight per day. The search area per day is therefore:

$$93km^2/h \cdot 10h/d = 930km^2/d \tag{3}$$

4 Moonbird

Sea-Watch e.V. and the Human Pilot Initiative (HPI) operate a Cirrus SR22 plane which is located in Malta. The top speed 185 kts (=340 km/h). The maximum operational range including safety margin is 1108 nautical miles (2050 km) at 154 kts (= 285 km/h) corresponding to a flight duration of 7,1 hours [2, p. 5-26]. The distance from Malta to the search area is 300 km. So the plane is travelling 2,1 hours to and from the search area and has LS = 1452 km or 5 hours left in the search area for search operations. The covered search area per hour is therefore

$$\frac{A}{t} = Wv = 4,3km \cdot 285km/h = 1225km^2/h \tag{4}$$

The total search area for one flight is given in equation 5.

$$A = W \cdot LS = 4,3km \cdot 1452km = 6243km^2 \tag{5}$$

One flight costs about 2500 Euros. One flight per day ist realistic, i.e that is the coverage per day.

5 SearchWing UAV

The searchwing UAV has a cruise speed of approx. 50 km/h.

5.1 Camera System

The camera is the Raspberry Pi Camera V2.1 [3]. The specs are given in table 3.

Table 3: Technical parameters of Raspberry Pi V2.1 Camera

Parameter	Value
sensor	Sony IMX219
resolution	3280 x 2464 pixels
optical size	1/4" (1 inch = 25.4 mm, 1/4 inch = 6.35 mm)
sensor size	3.68mm x 2.76mm (diagonal 4.6 mm)
horizontal field of view	62.2°
vertical field of view	48.8°
focal length	3.04 mm

For objects which are far away, the image is displayed at the focal length of the lens. The horizontal and vertical field of view can therefore be computed from the sensor dimensions and the focal length of the camera.

$$hfov = 2arctan(\frac{sw}{2fl}) = 2arctan(\frac{3.68mm}{2 \cdot 3.04mm}) = 62.2° \tag{6}$$

$$vfov = 2arctan(\frac{sh}{2fl}) = 2arctan(\frac{2.76mm}{2 \cdot 3.04mm}) = 48.8° \tag{7}$$

5.2 Camera looking straight down

For a camera that points straight down, the width on the ground is given by the flight height h and the horizontal field of view. Figure 2 depicts the width of the image with a flight altitude h. The angle α is derived from the horizontal field of view (62.2°/2 = 31.1°). This width w covers the complete horizontal sensor area, i.e. the full 3280 pixels.

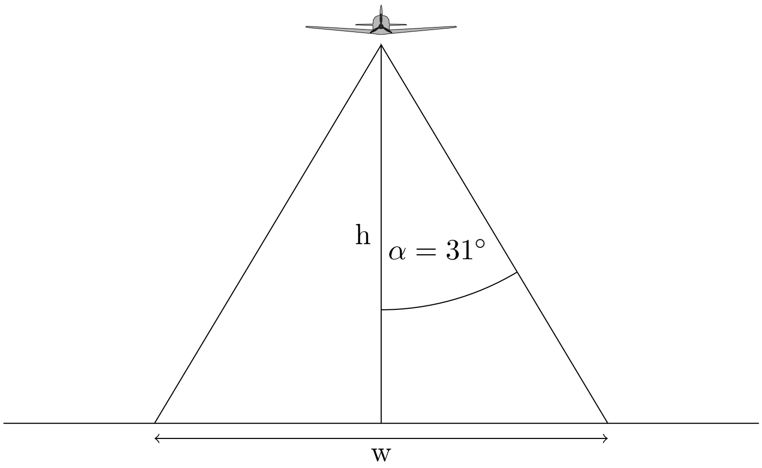

Figure 2: Raspberry Pi Camera V2.1 pointing down - width w at ground

The width w, the length l and the ground resolution g is now a function of the flight altitude h.

$$w = 2 \cdot tan(\alpha) \cdot h = 2 \cdot tan(31°) * h = 1.2 * h \tag{8}$$

$$l = 2 \cdot tan(\beta) \cdot h = 2 \cdot tan(24.4°) * h = 0.9 * h \tag{9}$$

$$g = w/3280px = 2 \cdot tan(\alpha) \cdot h/3280 \tag{10}$$

Table 4 show the image size on the ground and the ground resolution for the Rasperry Pi Cam v2.1. w is the image width and l is the image height. h ist the flight altitude of the plane above ground.

Table 4: Ground image for Raspberry Cam v2.1

h / m	w / m	l / m	Ground resolution cm/px
100	120	90	3.66
150	180	135	5.5
550	660	495	20
800	960	720	29
1100	1320	990	40

Table 4 is based on the default raspberry pi camera v2.1 parameters, i.e. a horizontal field of view of 62°. If the raspberry camera would be modified with another lens, e.g. an M12 lens, then the FOV is changed and the table above looks different.

5.3 Camera looking to the side

If the camera does not look directly down to the sea surface, then the ground resolution changes in the image due to the perspective. Figure 3 shows the scenario of a raspberry pi camera with a horizontal field of view of 62°. The camera looks $\gamma = 40°$ to the side. The ground resolution at point A is higher than the ground resolution at point B which is further away from the camera. Point V is an example point in the field of view of the camera. Point V has an inclination of angle ϕ with respect to the direct line to the ground OD. The angle with respect to the view angle of the camera is β.

Each pixel on the ground corresponds to a an image plane with a different distance between point on the ground and camera. An example image plane is shown for point M. The image plane is perpendicular to the OM line. For point V the image plane is closer to the camera and is defined by the line VS. Table 6 explains the different points, angles and distances in figure 3.

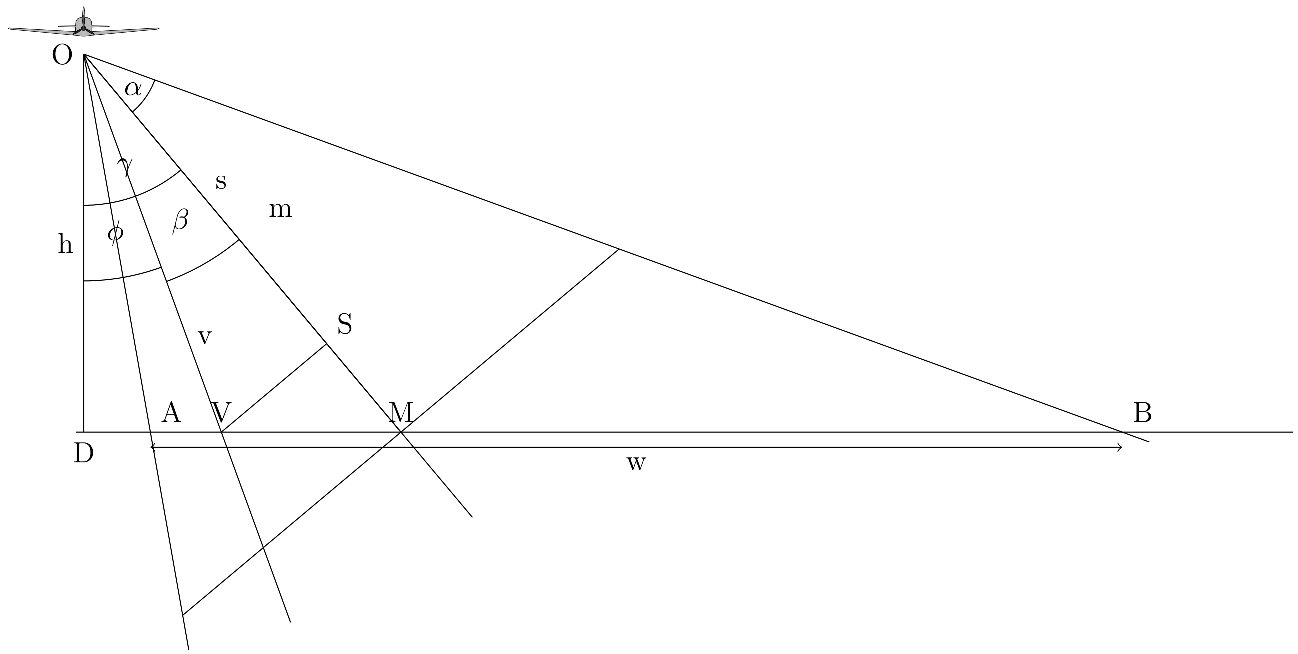

Figure 3: Raspberry Pi Camera V2.1 looking 40 degrees to the side

Table 5: Explanation of markers in the figure

Name	Comment
O	Position of the camera at the plane
D	Position on the ground directly below the plane
M	Intersection of the ground and the direct view from camera
A	Closest point to D shown on camera. Highest resolution
B	Last point shown on camera. Lowest resolution
V	Example view point on camera image
S	Intersection of camera view angle and mage plane defined by view point V
w	Distance between A and B is the visible horizontal width for the camera
h	Distance OD. Altitude of the camera above ground
m	Distance OM between horizontal middle pixel and camera
v	Distance OV between camera and view point V
s	Distance OS from image plane for point V and camera
γ	View angle of the camera, i.e. the tilt of the camera
α	Half the horizontal field of view, i.e. 31 degrees for the Pi camera
ϕ	Angle for the view point V
β	$\phi + \beta = \gamma$

The ground resolution is different for each pixel and is defined by the image plane which intersects with the point on the ground. The closer the image plane is to the camera, the higher is the resolution on the ground. The resolution for the image plane can be calculated according to the previous chapter. The modification is that the relevant distance between camera and image plane is OS=s for point V in the example of figure 3. V is defined by angle ϕ. So s is a function of V and hence ϕ. Equation 11 shows the ground resolution for point V.

$$g(V) = 2 \cdot tan(\alpha) \cdot s/3280 \tag{11}$$

The length s can be calculated when distance OV = v is known.

$$cos(\phi) = \frac{h}{v} \Rightarrow v = \frac{h}{cos(\phi)} \tag{12}$$

$$cos(\beta) = \frac{s}{v} \Rightarrow s = vcos(\beta) \tag{13}$$

$$\Rightarrow s = \frac{h \cdot cos(\beta)}{cos(\phi)} \tag{14}$$

$$\beta = \gamma - \phi \tag{15}$$

$$\Rightarrow s = \frac{h \cdot cos(\gamma - \phi)}{cos(\phi)} \tag{16}$$

Equation 16 gives the distance s from the camera to the image plane for point V in the figure. Taking equation 16 for s together with equation 11 we can calculate the ground resolution for point V when V is defined by the angle ϕ.

$$g(V) = 2 \cdot tan(\alpha) \cdot \frac{h \cdot cos(\gamma - \phi)}{3280 \cdot cos(\phi)} \tag{17}$$

The width w of the horizontal camera view ist the distance between the points A and B. Point V becomes point A when $\phi = \gamma - \alpha$. Point V becomes B when $\phi = \gamma + \alpha$. The distance DV is given by equation 19

$$tan(\phi) = \frac{DV}{h} \tag{18}$$

$$\Rightarrow DV = h \cdot tan(\phi) \tag{19}$$

Therefore the width w can be computed from the difference of DB and DA according to equation 22.

$$DA = h \cdot tan(\gamma - \alpha) \tag{20}$$

$$DB = h \cdot tan(\gamma + \alpha) \tag{21}$$

$$w = DB - DA = h(tan(\gamma + \alpha) - tan(\gamma - \alpha)) \tag{22}$$

Table 6 shows the angles, resolutions and distances for different angles angles and altitudes. Note that the ground resolutions and the distances scale with altitude. The distances and the resolution is given in meter. The angles are given in degrees. For example the line with altitude 550m and camera inclination angle γ of 40 degrees shows that point A has an angle ϕ of 9 degrees and point B 71 degrees. At point A the ground resolution is 17.5 cm/pixel. At point M the ground resolution is 26.3 cm/pixel and at point B the resolution is 53.1 cm/pixel. The distance of point D to A is 87m and the distance of D to B is 1597m. This gives an observed image width of 1510m. If there were two cameras on the plane where on would look to the right and the other looks to the left, then a gap of 2 * 87m = 174m directly below the plane would not be observed by both cameras.

With inclination of 31 degrees and two cameras where on looks to the left and the other to the right, the distance to point A is zero, i.e. there would be no gap. The width for each camera would be 1034m and the worst resolution at point B is 36.8 cm/pixel. This would give a total image width of 2068m.

Table 6: Resolutions and distances for camera looking to the side

Altitude	γ	ϕ for A	ϕ for B	Res. A	Res. M	Res. B	DA	DM	DB	w
100	31	0	62	0.031	0.043	0.067	0.0	60	188	188
	35	4	66	0.031	0.045	0.077	7	70	225	218
	40	9	71	0.032	0.048	0.096	16	84	290	275
	45	14	76	0.032	0.052	0.130	25	100	401	376
	50	19	81	0.033	0.057	0.201	34	119	631	597
	55	24	86	0.034	0.064	0.450	45	143	1430	1386
	58	27	89	0.035	0.069	1.799	51	160	5729	5678
550	31	0	62	0.173	0.235	0.368	0	330	1034	1034
	35	4	66	0.173	0.246	0.425	38	385	1235	1197
	40	9	71	0.175	0.263	0.531	87	461	1597	1510
	45	14	76	0.178	0.285	0.714	137	550	2205	2069
	50	19	81	0.183	0.313	1.104	189	655	3473	3283
	55	24	86	0.189	0.351	2.476	245	785	7865	7620
	58	27	89	0.194	0.380	9.897	280	880	31509	31229

5.4 Search Area

The current camera looks straight down and will cover a width of 660m with a ground resolution of 20cm/pixel at an altitude of 550m according to table 4. I assume that this corresponds to the sweep width of the system, i.e. every boat will be detected in this 660m wide corridor. Each flight may have a distance of 100km. Within the range of approx. 4km around the launching vessel, any boat will be detected by the crew anyway. Therefore I subtract 8 km for each flight. The effective distance is therefore 100km - 8km = 92km. With a sweep width of 660m, each flight will cover a useful search area of

$$A_{flight} = sw \cdot d = 0.66km \cdot 92km = 60km^2 \tag{23}$$

Assuming a cruise speed of 50 km/h each flight will take around 2 hours. Assuming 10h daylight operation and zero time for battery change, this will allow 5 flights per day. Overall this will allow to search an area of $5 \cdot 60km^2 = 300km^2$ per day.

Compared to the Moonbird (4.3 km) and the Seawatch (9.3 km) the sweep width of 660m of the UAV is small. The total search area will scale directly with this sweep width, i.e. doubling the sweep width from 660m to 1320m will double the search area to 600 km^2.

6 Search Theory

The International Aeronautical and Maritime Search and Rescue Manual [1], gives guidelines for search and rescue operations. The theory of search gives a model of detecting an object which is based on the probability of detection (POD) which depends on the distance between the object and the sensor. One model derived in the 40s is the Koopmann inverse cube model [4, Section 4.6] [5, p. 21]. The inverse cube model is based on the following model idea.

a) the search objects are warships

b) the search is done with an aircraft with height h above the ocean

c) the ship is detected by the observer in the aircraft sighting the ships wake

d) the instantaneous (one glimpse) probability γ of sighting the ships wake is proportional to the solid angle subtended at the observers eye by the wakes visible area

Figure 6 shows the scenario for the inverse cube model. The observer at location O is distance h above the ocean. The length a is the objects surface length in this dimension at the sea. The model is that this is wake, i.e. the object is flat on the sea surface. The assumption is that angle α is small, i.e. that the area covered by the object is small in comparison to the total field of view. The total area on the surface is $A = ab$. The visible area from the observer is $A_v = cb$.

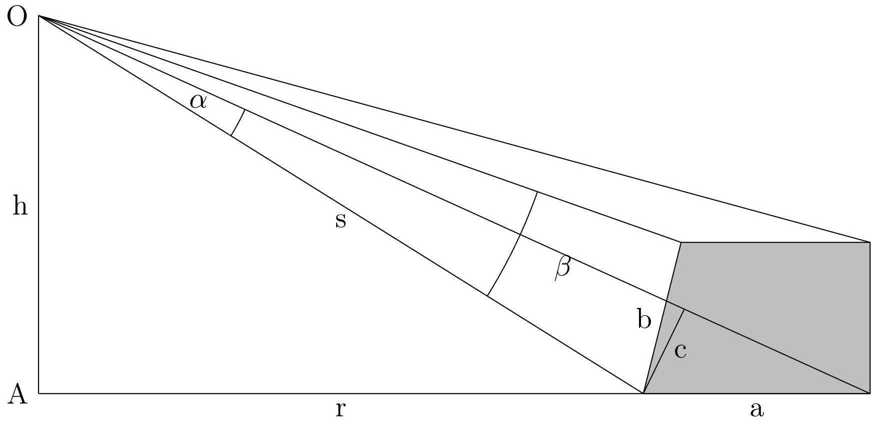

Figure 4: Inverse cube model - Observing the ocean from above

For small angles, the angle in radian can be approximated. If a is small compared to h and r, then the triangles become similar and equation 24 becomes valid. Therefore α can be expressed in terms of h and s.

$$\alpha = \frac{c}{s}, \beta = \frac{b}{s}, \frac{c}{a} = \frac{h}{s} \tag{24}$$

$$\alpha = \frac{c}{s} = \frac{ha}{s^2} \tag{25}$$

The solid angle Ω, i.e. the angle area in the field of view is the product of α and β as described in 26. The assumption is that the glimpse probability to detect the object γ is proportional to Ω. Therefore the instanteneous probability is as described in equation 27.

$$\Omega = \alpha\beta = \frac{hab}{s^3} \tag{26}$$

$$\gamma = k\Omega = k\alpha\beta = k\frac{hab}{s^3} = k\frac{hA}{(h^2 + r^2)^{\frac{3}{2}}} \tag{27}$$

The constant k describes all other factors that are relevant for detection like color or contrast. Equation 27 is called Koopmans inverse cube law. With this rule, the chance to detect an object is proportional to the size of the object but it reduces with the inverse of the cube of the distance between observer and object. Figure 6 sketches the inverse cube model for the instantenous detection probability of an object.

During the search operation there are several opportunities to detect the object. For a human visual search this corresponds to a fixation period of the human eye of about 250 ms. Assuming a non moving ship S and an observer search aircraft with a velocity w travelling a straight line, then

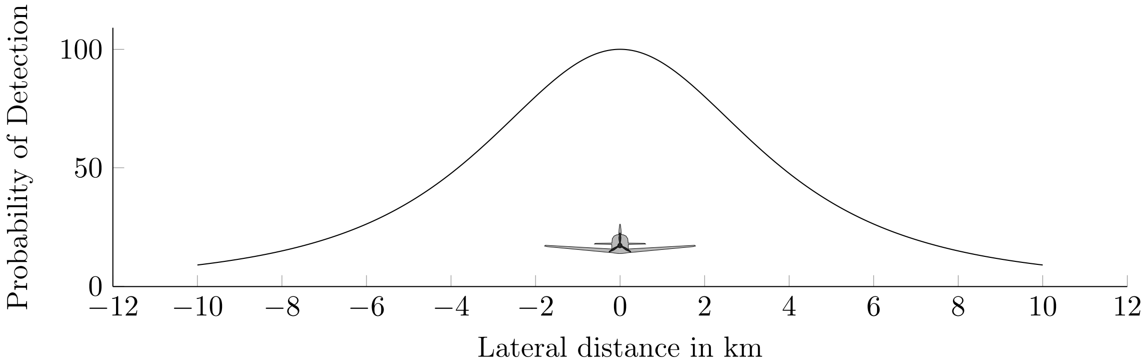

Figure 5: Koopmans Inverse Cube Model - Probability to detect an object in one glimpse as a function of size and distance

the aircraft will pass the object with minimum distance x_{min} at a point O_{min}. However on the whole path there is possibility to detect the ship S, although at other positions O, the distance between ship and observer is larger than x_{min}.

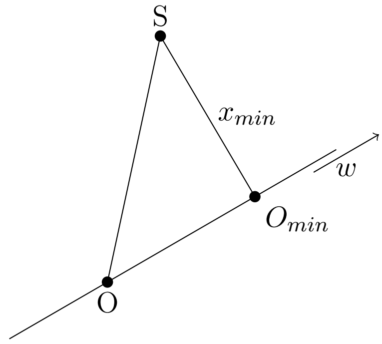

Figure 6: Oberser travels on a straight path with velocity w with minimum distance to ship S equal to x_{min}

The total probability to detect the ship is the integral of the instanteneous probability γ over the whole path. The result is given in equation 28 [5]

$$p(x) = 1 - e^{-\frac{2kh}{wx^2}}; \tag{28}$$

Figure 7 shows the resulting probability of detection when an observer travels a straight line passing an object at minimum distance x. This does not mean that the object is detected at the minimum distance. The object might be detected at any point on the line but when the distance increases, the probability decreases.

The curve in figure 7 is an example of a lateral range curve. The curve basically depends on the sensors characteristics. Another lateral range curve is the definite range curve depicted in figure 8. Every object in a range of R = 4 km will be detected with 100% certainty. All objects with a higher distance than 4 km will definitely not be detected.

In order to compare sensors with different characteristics, the sweep width W is introduced. When travelling over an area with an equal random object distribution, sensors with the same sweep width will find the same number of objects on average when they have the same sweep width W. Sensors with a lower sweep width will find less objects. The sweep width is defined as the integral over the

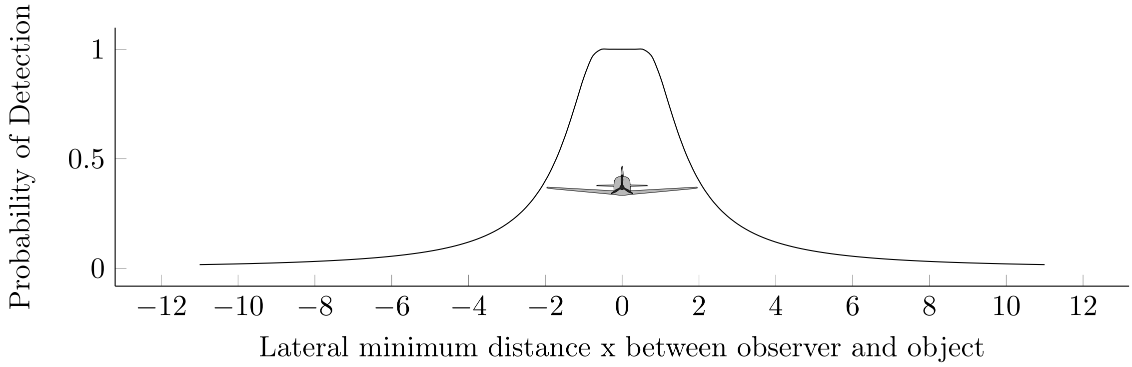

Figure 7: Probability to detect an object that has a minimum distance x from the observer

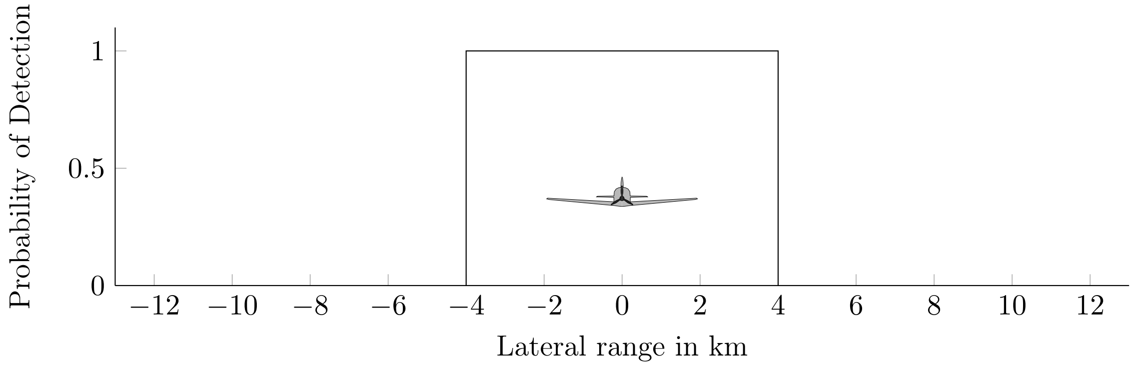

Figure 8: Definite Range Curve

lateral range curve. Therefore the sweep width of the lateral range type sensor is

$$W = 2 * R \tag{29}$$

The inverse cube model type sensor has a sweep width of

$$W = \int_{-\infty}^{\infty} p(x)dx = \int_{-\infty}^{\infty} 1 - e^{-\frac{2kh}{wx^2}} dx = 2\sqrt{\frac{2\pi kh}{w}} \tag{30}$$

which predicts an unlimited increase of sweep width when just the height of the plane is increased. This is obviously not correct. Figure 9 shows a definite range sensor and an inverse cube model type sensor which have both the same sweep width W of 5km.

7 Conclusion

Comparing the search areas that are covered by Moonbird, Seawatch 3 and Searchwing drone shows remarkable differences. Sweep width and speed define the raw search area per hour. However the Moonbird starts in Malta and so only part of the time is accounted to search. For the Searchwing drone the area within 4km of the launching vessel does not count. The analysis shows that the performance of the drone can be increased substantially by increasing the sweep width.

The discussion in the chapter search theory shows that it is not required to detect every boat in the image but that there might be an optimum when the image width is increased even when not all boats are detected. The relevant parameter is the detection probability which will decrease with

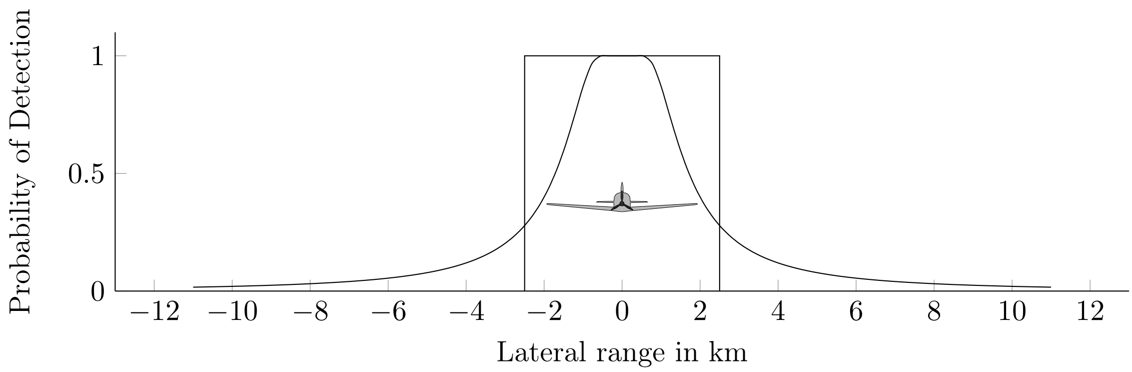

Figure 9: Definite Range and Inverse Cube Sensors with equal sweep width W = 5 km

Table 7: Comparison of search vehicles

Searcher	Sweep Width km	Speed km/h	Raw area per h km^2/h	Adj. area per day km^2/d	Remark
Moonbird	4.3	285	1225	6243	Start in Malta
Seawatch 3	9.3	10	93	930	10h per day
Searchwing	0.66	50	33	303	100km per flight
Searchwing	2	50	100	920	Inc. sweep width

distance from the drone. However, maybe the equivalant sweep width might be larger compared to a pretty small image width where 100% of all boats are detected. This is illustrated in figure 9.

Sentientvision offers a camera device including image recognition which can be attached to a Boing ScanEagle UAV which has wingspan of 3.1m and a mass of 20kg [6]. According to the datasheet a six person liferaft can be detected within a range of 3.5 nautical miles, i.e. 6.5 km. If this detection is very probable, then this would yield a sweep width of 13 km. The Vidar is equipped with a 9 megapixel camera which is attached to a gimbal that scans the heading 180 degrees. This indicates that there is some room for improvement regarding the camera and image processing.

References

1. IOM. *IAMSAR Manual*. 2007.
2. CIRRUS. *Pilot Handbook Cirrus SR22*. Cirrus Design Corporation, 2011. Available also from: `http://www.aeroprague.com/cz/download/Cirrus_SR22_Handbook_and_Manual.pdf`.
3. RASPBERRY. *Raspberry Pi Camera V2*. Raspberry, 2011. Available also from: `https://www.raspberrypi.org/documentation/hardware/camera/`.
4. OFFICE OF SEARCH AND RESCUE - U.S. COAST GUARD. *The Theory of Search*. 1996. Available also from: `https://www.navcen.uscg.gov/pdf/Theory_of_Search.pdf`.
5. KOOPMAN, Bernard Osgood. *Search and Screening - OEG Report 56*. Operations Evaluation Group - U.S. Navy, 1946. Available also from: `https://www.informs.org/Explore/History-of-O.R.-Excellence/Documents/Bernard-O.-Koopman-Search-and-Screening-1946`.
6. *Sentientvision Visual Radar* [online]. Sentientvision [visited on 2019-06-06]. Available from: `http://www.sentientvision.com/products/vidar-2-2/`.